An approximate transfer function for the dual-resonance nonlinear filter model of auditory frequency selectivity

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The dual-resonance nonlinear filter [Meddis *et al.*, J. Acoust. Soc. Am. **109**, 2852–2861 (2001)] was presented as a digital time-domain algorithm to model nonlinear auditory frequency selectivity. This report extends previous work by presenting an approximate analytic transfer function that allows calculating and analyzing its level-dependent frequency-domain response. The transfer function is derived on the assumption that the filter behaves linearly for any given input amplitude. It matches accurately the response (gain and phase) of the digital filter for tones. Practical uses for the transfer function are suggested. © *2003 Acoustical Society of America.* [DOI: 10.1121/1.1605389]

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I. INTRODUCTION

The dual-resonance nonlinear (DRNL) filter is an algorithm capable of reproducing an important number of physiological (Meddis *et al.*, 2001; Sumner *et al.*, 2002, 2003a, 2003b) and psychophysical (Lopez-Poveda and Meddis, 2001) phenomena pertaining to nonlinear auditory frequency selectivity. It was originally designed digitally in the *time* domain to extend its validity for complex, broadband stimuli and to facilitate its application to real-time systems. However, filters are better understood in terms of their *frequency*-domain response. This is particularly true for the DRNL filter as its best frequency, tuning, gain, and phase characteristics change with the amplitude of its input signal. This report extends previous work by presenting an *analytic* transfer function for evaluating the amplitude-dependent frequency response (gain and phase) of the DRNL filter based on its response to *tones*.

Nonlinear filters alter the spectral content of the input waveform, and thus do not have proper transfer functions (Smith, 2002). Indeed, deriving an approximate transfer function of the DRNL filter has been possible because all but one of its components (Fig. 1) are linear and time invariant. The nonlinear stage [$U(f)$ in Fig. 1] applies a memoryless, time-*varying* gain that depends on the instantaneous amplitude of its input waveform. Although its exact transfer function does not exist, an approximation is made here to obtain it. It consists of treating the nonlinear stage as a time-*invariant* gain that depends on the *peak* amplitude of its input. This approximation is equivalent to assuming that the DRNL filter is *linear* for any given input level. Below it is shown that this approximation preserves, with good accuracy, the nonlinear gain and phase properties of the digital DRNL filter in response to tones, although it sacrifices the effect of the original nonlinear gain stage on the spectral content of its output.

The proposed transfer function may have a number of applications. The assumption on which it has been derived (linear behavior of the filter for any given input level) is commonly made in psychophysics for measuring auditory filter shapes (Moore, 1998, Chap. 3), and in physiology for measuring basilar-membrane (BM) isointensity and input/output curves (Robles and Ruggero, 2001). Therefore, it may be used to model these types of data. The transfer function demonstrates the contribution of every parameter of every component stage of the DRNL filter to its frequency response. This knowledge facilitates tuning the numerous parameters of the filter to reproduce specific data sets.

The transfer function may also be used to investigate the extent that suppression and distortion phenomena affect auditory filter shapes derived from notch-noise data (Moore, 1998). An estimate could be obtained by comparing the transfer function against the response of the digital DRNL filter to the notch-noise stimuli, both computed with identical parameters tuned for the transfer function to match the filter shapes. The difference in response may be attributed to suppression and distortion present during data collection, as these effects will be modeled by the digital DRNL filter (Meddis *et al.*, 2001) but not by the transfer function.

The transfer function allows computing the response of the DRNL filter for tones much more rapidly than its digital implementation. Therefore, it may be particularly useful for applications that require evaluating the frequency response of filter *banks*; for example, during the development of speech processing strategies for auditory prostheses based on the DRNL filter (e.g., Wilson *et al.*, 2002). It may also be used to compute level-dependent excitation patterns from auditory filter shapes modeled with the DRNL filter using the method of Glasberg and Moore (1990).

II. THE TRANSFER FUNCTION

The transfer function of a filter is the ratio of the Fourier transform of its output signal, $y(t)$, to the Fourier transform of the input signal, $x(t)$ (Hartmann, 1998, p. 195). For the DRNL filter (Fig. 1), this can be written as

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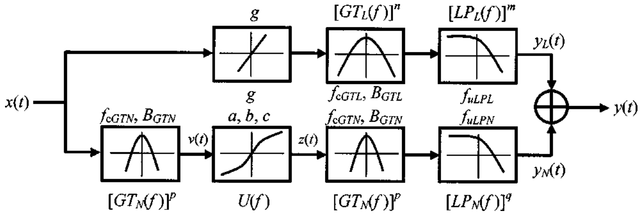

FIG. 1. Architecture of the dual-resonance nonlinear filter. The symbols between the top and the bottom processing paths inform of the parameters of each filter stage. The transfer function of each (GT or LP) filter or gain stage is given above (top path) or below (bottom path) its corresponding block. See the main text for details.

$$H_{\mathrm{DRNL}}(f)=\frac{\mathcal{F}[y(t)](f)}{\mathcal{F}[x(t)](f)}, \quad (1)$$

where $\mathcal{F}$ means Fourier transform. The output from the DRNL filter is the sum of the outputs from its linear, $y_L(t)$, and nonlinear, $y_N(t)$, paths. The input to both paths is the same and equal to $x(t)$. Therefore, by virtue of the linear properties of the Fourier transform, Eq. (1) can be rewritten as

$$H_{\mathrm{DRNL}}(f)=\frac{\mathcal{F}[y_L(t)+y_N(t)](f)}{\mathcal{F}[x(t)](f)}=H_L(f)+H_N(f). \quad (2)$$

That is, the transfer function of the DRNL filter can be expressed as the sum of the individual transfer functions of its two paths, $H_L(f)$ and $H_N(f)$.

A. The transfer function of the linear path

The linear path (Fig. 1) consists of a linear gain, *g*, followed by a cascade of *n first*-order gammatone (GT) filters followed by a cascade of *m second*-order Butterworth low-pass (LP) filters, all with unit gain in their passbands. All these elements are linear and time invariant. Therefore, the transfer function of the linear path, $H_L(f)$, can be expressed as the product of the individual transfer functions of its three stages

$$H_L(f)=g\cdot[GT_L(f)]^n\cdot[LP_L(f)]^m, \quad (3)$$

where $GT_L(f)$, and $LP_L(f)$ denote the transfer functions of the individual GT and LP filters in the linear path, respectively. These are described in the following two sections.

1. The transfer function of the first-order gammatone filter

The transfer function of a *first*-order GT filter with unit gain at its center frequency, f_c (Hz), is as follows (Stone, 1995; Hartmann, 1998):

$$GT(f)=k\cdot\left[\frac{B}{B+j(f-f_c)}+\frac{B}{B+j(f+f_c)}\right], \quad (4)$$

where $j=\sqrt{(-1)}$, B is *half* the 3-dB bandwidth (Hz) of the *first*-order GT filter (Hartmann, 1998), and k is a constant that makes $GT(f)$ have unit gain at f_c

$$k=\frac{1}{|1+(B/B+2jf_c)|}. \quad (5)$$

For convenience, $GT(f)$ can be expressed using complex polar notation as

$$\mathrm{GT}(f)=R_{\mathrm{GT}}(f)\cdot\exp[j\phi_{\mathrm{GT}}(f)], \quad (6)$$

where $R_{\mathrm{GT}}(f)=|\mathrm{GT}(f)|$ is the gain of the GT filter, and $\phi_{\mathrm{GT}}(f)=\arg[\mathrm{GT}(f)]$ its phase.

2. The transfer function of the second-order Butterworth low-pass filter

The transfer function of a second-order Butterworth low-pass filter, $\mathrm{LP}(f)$, can be expressed using complex polar notation as

$$\mathrm{LP}(f)=R_{\mathrm{LP}}(f)\cdot\exp[j\phi_{\mathrm{LP}}(f)] \quad (7a)$$

(Oppenheim *et al.*, 1999), where the gain, $R_{\mathrm{LP}}(f)$, and the phase, $\phi_{\mathrm{LP}}(f)$, of the filter are

$$R_{\mathrm{LP}}(f)=\left[\frac{1}{(1-f_R^2)^2+2f_R^2}\right]^{0.5}, \quad (7b)$$

$$\phi_{LP}(f)=\arctan\left(\frac{-\sqrt{2}f_R}{1-f_R^2}\right), \quad (7c)$$

with $f_R=f/f_u$, and f_u being the 3-dB-down cutoff frequency (Hz) of the filter.

Therefore, the transfer function of the linear path can be obtained by substituting Eqs. (6) and (7a) into Eq. (3)

$$H_L(f)=g\cdot[R_{\mathrm{GTL}}(f)]^n\cdot[R_{\mathrm{LPL}}(f)]^m\cdot\exp\{j[n\phi_{\mathrm{GTL}}(f)+m\phi_{\mathrm{LPL}}(f)]\}. \quad (8)$$

The subscripts GTL and LPL refer to the GT and LP filters in the linear path, respectively.

B. The transfer function of the nonlinear path

The nonlinear path (Fig. 1) is a cascade of *p* first-order GT filters, followed by a *compressive* nonlinear gain, followed by another cascade of *p* GT filters (identical to the first one), followed by a cascade of *q* second-order Butterworth LP filters. Assuming that the transfer function for the nonlinear gain, $U(f)$, exists (see below), the transfer function of the nonlinear path can be expressed as the product of the transfer functions of its stages

$$H_N(f)=U(f)\cdot[\mathrm{GT}_N(f)]^{2p}\cdot[\mathrm{LP}_N(f)]^q, \quad (9)$$

where $\mathrm{GT}_N(f)$ and $LP_N(f)$ denote the transfer functions of the individual GT and LP filters in the nonlinear path, respectively.

Only the transfer function of the nonlinear gain, $U(f)$, remains to be described.

1. The transfer function of the nonlinear gain

Meddis *et al.* (2001) defined the time-domain form of the nonlinear gain as

$$z(t)=\mathrm{sign}[v(t)]\cdot\min\{a\cdot|v(t)|,b\cdot|v(t)|^c\}, \quad (10)$$

where $v(t)$ and $z(t)$ are the input and output waveforms to/from the nonlinear gain (Fig. 1), a and b are gain parameters ($\geqslant 0$), and c is the compression exponent ($0<c\leqslant 1$). It is, therefore, a memoryless (instantaneous), time-*varying*

nonlinear gain. As such, it is *not* possible to derive its analytic transfer function (Smith, 2002).

In practice (e.g., Meddis *et al.*, 2001; Lopez-Poveda and Meddis, 2001), the frequency response of the *digital* DRNL filter is evaluated by examining the *peak* amplitude and the phase of its output waveform in response to sinusoids. For sinusoidal inputs, an approximation can be made that allows derivation of an analytic transfer function for the nonlinear-gain stage. Let $u(t)$ denote the approximated time-domain function, which is as follows:

$$u(t)=\min(a,bV^{c-1})\cdot v(t). \tag{11}$$

$u(t)$ applies an instantaneous time-*invariant* gain that depends only on the *peak* amplitude, V, of the input sinusoid, $v(t)$. The error of the approximation is zero when V is less than or equal to the compression-threshold amplitude $V_c=(b/a)^{1/(1-c)}$, as $u(t)=z(t)$. However, when $V>V_c$ the original nonlinear gain, $z(t)$, applies instantaneous compression that alters the shape of the input waveform [Fig. 2(a)] and hence its spectral content [Fig. 2(b)]. This property is *not* preserved by the approximated gain, $u(t)$, which remains a pure sinusoid. Nevertheless, the frequency and phase of $u(t)$ are identical to those of the fundamental frequency of $z(t)$. Furthermore, its amplitude is equal to the *peak* amplitude of $z(t)$ [Fig. 2(a)]. In summary, $u(t)$ maintains the properties of the original nonlinear gain, and hence of the DRNL filter, regarding the gain and the phase of its response to sinusoids (see Sec. III below).

An important advantage of Eq. (11) is that it has an exact transfer function

$$U(f)=\min(a,bV^{c-1}). \tag{12}$$

In the DRNL filter (Fig. 1), V is the product between the peak amplitude of the input sinusoid to the DRNL filter, X, and the gain of the first GT-filter cascade in the nonlinear path, $[R_{\mathrm{GTN}}(f)]^p$. Hence, V depends on the frequency *and* on the peak amplitude of the input tone to the DRNL filter

$$V(f,X)=X\cdot[R_{\mathrm{GTN}}(f)]^p. \tag{13}$$

The transfer function of the approximated nonlinearity is obtained by replacing $V(f,X)$ into Eq. (12)

$$U(f,X)=\min\{a,b[R_{\mathrm{GTN}}(f)]^{p(c-1)}X^{c-1}\}. \tag{14}$$

The notation $U(f,X)$ makes explicit that the transfer function depends on the frequency of the input tone to the DRNL filter *and* on its peak amplitude (X).

The frequency transfer function of the nonlinear path can now be derived by substituting Eqs. (6), (7a), and (14) into Eq. (9)

$$H_N(f,X)=\min\{a,b[R_{\mathrm{GTN}}(f)]^{p(c-1)}X^{c-1}\}\cdot[R_{\mathrm{GTN}}(f)]^{2p}\cdot[R_{\mathrm{LPN}}(f)]^q\cdot\exp\{j[2p\phi_{\mathrm{GTN}}(f)+q\phi_{\mathrm{LPN}}(f)]\}. \tag{15}$$

C. The transfer function of the DRNL filter

It follows from Eq. (2) that the transfer function of the DRNL filter, $H_{\mathrm{DRNL}}(f)$, can be expressed in polar form as

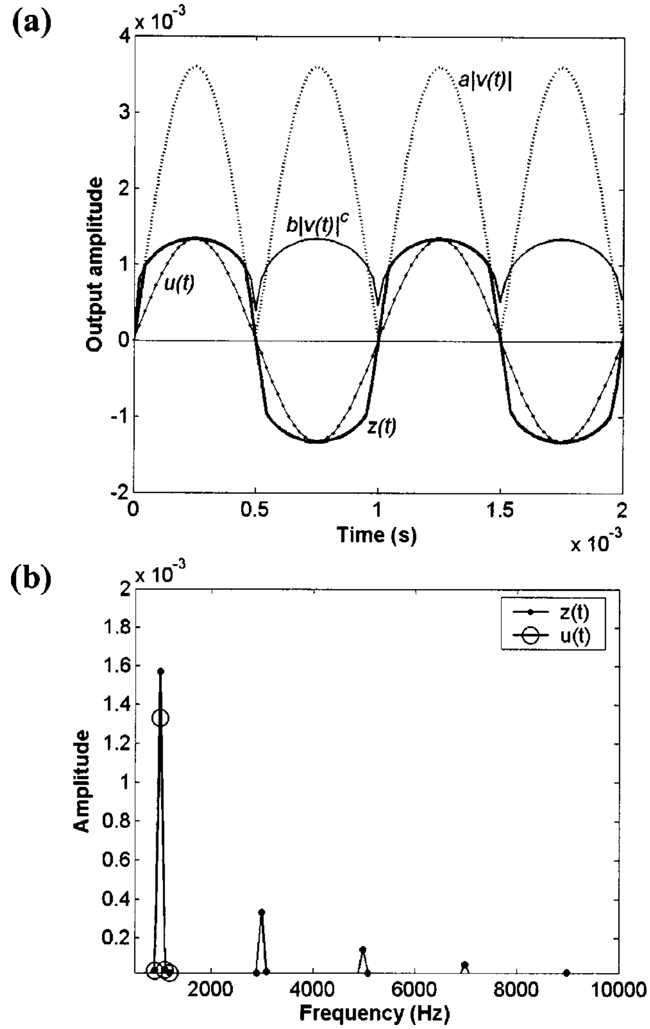

FIG. 2. (a) Comparison of the output waveforms from the original [Eq. (10)] and the approximated [Eq. (11)] nonlinear gains for an input waveform, $v(t)$, undergoing compression ($V>V_c$). The output waveform from the original nonlinear gain, $z(t)$ (thick continuous line), is equal to $av(t)$ at times when $a|v(t)|<b|v(t)|^c$, but equal to $b|v(t)|^c$ otherwise. It is periodical but not a pure sinusoid. Hence, it contains odd distortion harmonics [illustrated in panel (b)]. The output from the approximated nonlinear gain, $u(t)$, has the same peak amplitude *and* phase as $z(t)$, but is purely sinusoidal and hence shows *no* distortion harmonics [illustrated in panel (b)]. (b) Amplitude spectra of signals $z(t)$ and $u(t)$ shown in panel (a).

$$H_{\mathrm{DRNL}}(f)=R_{\mathrm{DRNL}}(f)\cdot\exp[j\phi_{\mathrm{DRNL}}(f)], \tag{16}$$

where

$$R_{\mathrm{DRNL}}(f)=[R_L^2+R_N^2+2R_LR_N\cos(\phi_N-\phi_L)]^{0.5}, \tag{17a}$$

$$\phi_{\mathrm{DRNL}}(f)=\arctan\left(\frac{R_L\sin\phi_L+R_N\sin\phi_N}{R_L\cos\phi_L+R_N\cos\phi_N}\right). \tag{17b}$$

$R_L(f)$ and $R_N(f)$ are the magnitudes of $H_L(f)$ and $H_N(f)$, respectively, and $\phi_L(f)$ and $\phi_N(f)$ their phases, all of which can be easily worked out from Eqs. (8) and (15).

It is noteworthy that Eqs. (17a) and (17b) show that when $R_L(f)\ll R_N(f)$, the gain and the phase of the DRNL filter are those of its nonlinear path. However, the linear path dominates the response when $R_L(f)\gg R_N(f)$. They also show that when $R_L(f)=R_N(f)$ *and* $(\phi_N-\phi_L)=N\pi(N=1,3,5,...)$, the gain of the DRNL filter is equal to zero and a notch occurs in the filter's frequency response, as noted by Meddis *et al.* (2001) and Lopez-Poveda and Meddis (2001).

III. EVALUATION

The validity of the transfer function was tested by comparing its output with the response of the digital, time-domain implementation[1] of the DRNL filter for identical sinusoidal inputs and for a large number of DRNL-filter parameter sets. The results shown in Fig. 3 are only an example. They are based on parameters reported in Meddis *et al.* (2001, Table I) for modeling basilar-membrane responses for the case L113 of Ruggero *et al.* (1997). These parameters are reproduced in Table I. The frequency of the sinusoids ranged from $0.25\times f_{c\mathrm{GTN}}$ to $2\times f_{c\mathrm{GTN}}$ in steps of $0.05\times f_{c\mathrm{GTN}}$. Their amplitude corresponded to sound-pressure levels ranging from 0 to 100 dB in steps of 20 dB, but it was scaled down to typical values of stapes velocity (in units of m/s), to match the expected order of magnitude for the DRNL-filter input. The scalar was fixed at 1.5×10^{-5} (m/s/Pa) across frequencies.

The amplitude and the phase responses of the digital DRNL filter were measured by applying a standard sine wave fit algorithm (Händel, 2000) to its output. The input sinusoids had a duration of 10 ms and were sampled at a rate of 10^6 Hz. The sampling rate was made so large to minimize any possible error due to sampling.

Figures 3(a) and (b) show a close match in the gain and the phase responses between the digital and the analytical implementations. Figure 3(c) shows that the error is small, particularly for low-and high-input amplitudes, where the DRNL filter behaves linearly.

Although not illustrated here, the discrepancy between the transfer function and the digital evaluations increases when lower sampling rates are used. The discrepancy is qualitatively more important in the phase response of the filter. It is attributed to sampling and is most prominent for the GT filters than for the low-pass filters, especially for high f_c's and for frequencies remote from f_c (see the footnote).

Increasing the amount of compression (by decreasing exponent *c* of the compressive nonlinearity) hardly reduces the accuracy of the transfer function. However, its match with the digital evaluation improves when *c* increases, as the filter behaves more linearly and the negative effects of the approximation diminish.

Remarkably, the transfer function took 0.16 s of CPU time to compute, whereas its digital counterpart took 30.4 s (both computed in MATLAB™ 6.5). Obviously, the long time required for evaluating the digital DRNL filter is the result of using an excessively high sampling rate. However, the digital implementation still took 5.5 s when the sampling rate was ten times smaller (10^5 Hz).

IV. AN EXAMPLE APPLICATION OF THE APPROXIMATE ANALYTIC TRANSFER FUNCTION: MODELING BASILAR-MEMBRANE RESPONSES TO PURE TONES

Previous dedicated reports have shown that the digital DRNL filter reproduces to a good approximation BM responses to pure tones (Meddis *et al.*, 2001; Sumner *et al.*, 2002, 2003b; Lopez-Najera *et al.*, 2003). For this purpose, they compared the *peak* amplitude of the output waveform

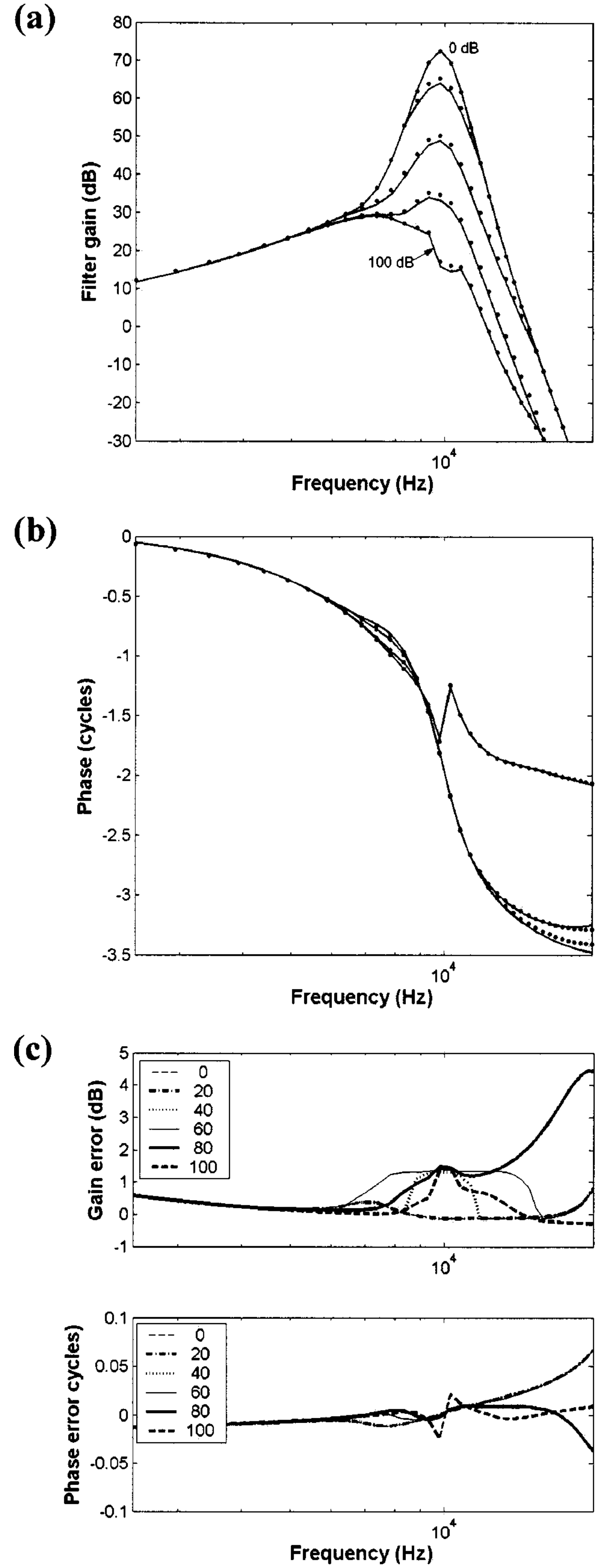

FIG. 3. A comparison between the response of the transfer function (continuous line) and the digital DRNL filter (dots). Different lines illustrate the response to different input amplitudes corresponding to levels ranging from 0 to 100 dB SPL in 20-dB steps. The amplitudes were further scaled down by 1.5×10^{-5} across frequencies (see the main text for details). (a) Gain (dB). Note the overlap between the lines for 0- and 20-dB input levels. (b) Phase (cycles). (c) Difference (digital minus transfer function) between the results obtained with the two methods.

TABLE I. Parameters for the DRNL filter used to produce the results shown in this report (reproduced from Table I of Meddis *et al.*, 2001, set L113).

Parameter	Value
Linear path	
n	2
m	4
g	100
$f_{c\text{GTL}}$ (Hz)	8 000
B_{GTL} (Hz)	2 000
$f_{u\text{LPL}}$ (Hz)	8 000
Nonlinear path	
p	3
q	3
a	12 000
b	0.057
c	0.25
$f_{c\text{GTN}}$ (Hz)	9 800
B_{GTN} (Hz)	1 400
$f_{u\text{LPN}}$ (Hz)	9 800

from the filter in response to pure tones of various frequencies and levels against corresponding experimental data (e.g., Meddis *et al.*, 2001). Above, it has been shown that the approximate analytic transfer function resembles the level-dependent frequency response of the digital filter as measured from its peak response to pure tones. Therefore, it may be used to model BM iso-intensity or input/output curves.

An example is shown in Fig. 4, where the analytic transfer function (thick continuous line) is compared against BM iso-intensity curves (thin dotted lines) reported by Ruggero *et al.* (1997, Fig. 9). The response of the digital DRNL filter is also shown (thick dashed lines) for comparison. Both the digital and analytic versions of the filter were computed as described in Sec. III. This time, however, the experimental frequency response of the stapes was used as the input (crosses in Fig. 4). This was taken from Fig. 9 of Ruggero *et al.* (1997) and was assumed to grow linearly with level. Both filter evaluations were computed only for those frequencies for which experimental data were available. Although no attempt was made to adjust the original parameters [provided by Meddis *et al.* (2001) to model this specific data set], the fit of the analytic transfer function is reasonable and comparable to that of the digital filter. The total Euclidean distance to the data was comparable for both evaluations: 57.1 dB for the digital, and 58.4 dB for the analytical version.

The procedure for optimizing the parameters of the approximate transfer function to model other data sets would be identical to that for the digital DRNL filter. The latter is described in detail elsewhere (Meddis *et al.*, 2001; Lopez-Poveda and Meddis, 2001). This may seem surprising at first, given that the digital filter is intrinsically nonlinear for each level, whereas the proposed transfer function is linear (but different for each level). However, it is noteworthy that the procedures of Meddis *et al.* (2001) and Lopez-Poveda and Meddis (2001) were based on adjusting the digital filter to reproduce experimental frequency responses in the form of input/output (or their equivalent iso-intensity) curves, such as those shown in Fig. 4. They then showed that other nonlinear properties (such as level-dependent frequency selectivity and phase, distortion, or suppression) *emerge* naturally from the filter as a result of its characteristic dual-resonance architecture. It follows from their procedure that the analytic transfer function may be used as a fast tool to optimize the parameters of the digital DRNL filter.

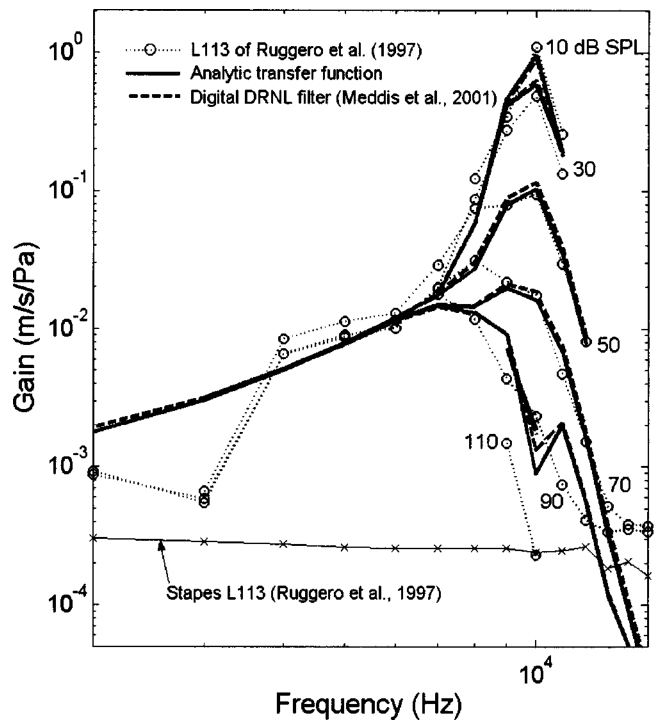

FIG. 4. A comparison of the approximate analytic transfer function (thick continuous line) with the iso-intensity gain BM data (dotted line, open circles) of Fig. 9 of Ruggero *et al.* (1997). The response of the digital DRNL filter is also shown (thick dashed line). Both versions of the DRNL filter were evaluated with identical parameters (given in Table I). They were computed for the same frequencies for which experimental data were available and for input levels ranging from 10 to 110 dB SPL in steps of 20 dB. These are indicated by the numbers next to each curve. Note that the fit of the analytic transfer function is comparable to that for the digital DRNL filter (see the main text for details).

It must be acknowledged, however, that Lopez-Najera *et al.* (2003) suggest that a more realistic nonlinear behavior is achieved when the parameters are optimized to fit *simultaneously* the *amplitude* and *phase* aspects of the BM response, and not only the amplitude aspect as Meddis *et al.* (2001) or Lopez-Poveda and Meddis (2001) did. The analytic transfer function is still valid for this purpose, as it also allows calculating the phase response of the digital DRNL filter.

V. DISCUSSION

The transfer function has been derived on an approximation (see above) that maintains the gain and the phase properties of the digital DRNL filter *almost* intact, but disregards the effect of the nonlinearity on the spectral content of its output [Fig. 2(b)]. That is, the transfer function does not preserve the distortion harmonics observed in output signal from the digital nonlinearity for a sinusoidal input. This defect is minimized by the fact that the second cascade of GT filters in the nonlinear path attenuates any high-order harmonics generated by the original nonlinearity. The degree of attenuation depends on the order and the bandwidth of these filters.

It is noteworthy that our approach for developing the approximate transfer function of the nonlinear gain, $U(f)$, does not require the particular form of the nonlinearity used in the DRNL filter. Any other nonlinear gain that does not deviate too much from a linear function over the range of input amplitudes of interest could also be ''linearized.''

The approximate transfer function does not preserve suppression and distortion phenomena characteristic of the digital DRNL filter when it is operating in its compression region. Therefore, it must be used with care for evaluating the output spectrum of the digital DRNL filter in response to *multitonal* or *broadband* stimuli whose peak amplitude (after allowing for filtering through the first GT cascade in the nonlinear path), V, exceeds the compression threshold amplitude, V_c. However, it provides accurate spectrum estimates for *any* stimuli such that $V < V_c$. For stimuli with very large amplitudes, generally such that $V \gg (g/b)^{1/(c-1)}$, the output from the linear path, R_L, is much larger than that from the nonlinear path, R_N. In this case, the DRNL filter behaves almost linearly and its approximate analytic transfer function also provides a good estimate of the output spectrum from the digital filter in response to *any* stimuli.

The DRNL filter, and more clearly its approximate analytic transfer function, suggest that auditory filters may be described by the added output from two parallel resonances, both intrinsically independent of level. It is the relative contribution of each resonance to the total filter output, controlled by its memoryless nonlinear gain, that confers the DRNL filter level-dependent frequency selectivity and gain. This scheme resembles de Boer's two-component EQ-NL theory (de Boer, 1997). According to this theory, the BM impedance is composed of two components, both intrinsically independent of level; one corresponds to the impedance of the ''passive'' BM, and one ''extra'' impedance that de Boer relates to the outer hair cells. The relative contribution of each component to the total BM impedance is controlled by a level-dependent factor whose role resembles that of the memoryless nonlinear gain of the DRNL filter. Interestingly, de Boer's theorem also asserts the existence of linear approximations to nonlinear auditory filters in certain circumstances (e.g., for wideband random noise of very low amplitude).

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[1]The digital DRNL filter was implemented in MATLAB™ as described in the Appendix of Lopez-Poveda and Meddis (2001). It uses digital IIR GT filters derived from the continuous-time GT impulse response by applying the impulse-invariance technique (Stone, 1995). Slightly different results from those shown in Fig. 3 would have been obtained if digital GT filters derived with the bilinear transformation had been used (R. Schatzer, personal communication, 2003). Slaney (1993) discusses different digital implementations of the GT filter. The bilinear GT filter provides a closer match to the *phase* of its transfer function for all frequencies and for the two sampling rates tested (10^5 and 10^6 Hz). Its *gain* is also closer to the transfer function for all conditions except for the lower sampling rate (10^5 Hz) *and* frequencies much higher than f_c, where the impulse-invariance GT filter performs better. MATLAB™ implementations of the digital DRNL filter and of its approximate transfer function are available from the author.

de Boer, E. (**1997**). ''Connecting frequency selectivity and nonlinearity for models of the cochlea,'' Aud. Neurosci. **3**, 377–388.

Glasberg, B. R., and Moore, B. C. J. (**1990**). ''Derivation of auditory filter shapes from notched-noise data,'' Hear. Res. **47**, 103–138.

Händel, P. (**2000**). ''Properties of the IEEE-STD-1057 four-parameter sine wave fit algorithm,'' IEEE Trans. Instrum. Meas. **49**, 1189–1193.

Hartmann, W. M. (**1998**). *Signals, Sound, and Sensation* (AIP, Springer, New York).

Lopez-Poveda, E. A., and Meddis, R. (**2001**). ''A human nonlinear cochlear filterbank,'' J. Acoust. Soc. Am. **110**, 3107–3118.

Lopez-Najera, A., Meddis, R., and Lopez-Poveda, E. A. (**2003**). ''A computational algorithm for computing nonlinear auditory frequency selectivity: Further studies,'' in Proceedings of the 13th International Symposium on Hearing, Dourdan, France.

Meddis, R., O'Mard, L. P., and Lopez-Poveda, E. A. (**2001**). ''A computational algorithm for computing nonlinear auditory frequency selectivity,'' J. Acoust. Soc. Am. **109**, 2852–2861.

Moore, B. C. J. (**1998**). *Cochlear Hearing Loss* (Whurr, London).

Oppenheim, A. V., Shafer, R. W., and Buck, J. R. (**1999**). *Discrete-time Signal Processing*, 2nd ed. (Prentice-Hall, Englewood Cliffs, NJ).

Robles, L., and Ruggero, M. A. (**2001**). ''Mechanics of the mammalian cochlea,'' Physiol. Rev. **81**, 1305–1352.

Ruggero, M. A., Rich, N. C., Recio, A., Shyamla Narayan, S., and Robles, L. (**1997**). ''Basilar membrane responses to tones at the base of the chinchilla cochlea,'' J. Acoust. Soc. Am. **101**, 2151–2163.

Slaney, M. (**1993**). ''An efficient implementation of the Patterson–Holdsworth auditory filter bank,'' Apple Computer Technical Report #35. Apple Computer Inc.

Smith, J. O. (**2002**). ''Introduction to Digital Filters,'' Center for Computer Research in Music and Acoustics (CCRMA), Stanford University. Web-published at http://www-ccrma.stanford.edu/~jos/filters/.

Stone, M. A. (**1995**). ''Spectral enhancement for the hearing impaired,'' Ph.D. dissertation. University of Cambridge, UK.

Sumner, C. J., Lopez-Poveda, E. A., O'Mard, L. P., and Meddis, R. (**2002**). ''A revised model of the inner hair cell and auditory-nerve complex,'' J. Acoust. Soc. Am. **111**, 2178–2188.

Sumner, C. J., Lopez-Poveda, E. A., O'Mard, L. P., and Meddis, R. (**2003a**). ''Adaptation in a revised inner-hair cell model,'' J. Acoust. Soc. Am. **113**, 893–901.

Sumner, C. J., Lopez-Poveda, E. A., O'Mard, L. P., and Meddis, R. (**2003b**). ''A nonlinear filter-bank model of the guinea-pig cochlear nerve,'' J. Acoust. Soc. Am. **113**, 3264–3274.

Wilson, B. S., Brill, S. M, Cartee, L. A., Cox, J. H., Lawson, D. T., Schatzer, R., and Wolford, R. D. (**2002**). ''Speech processors for auditory prostheses,'' Final report, NIH project N01-DC-8-2105.